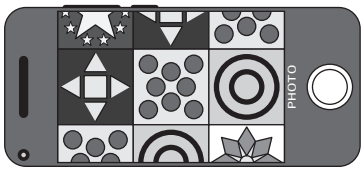
SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST

Benjamin

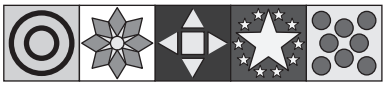
3 points

1 (Norway). A floor is made of 5 different tiles. The tiles are laid in a repeating pattern. Eva takes a picture of the floor with her phone, as shown. What is the repeating pattern of the 5 tiles?



(A) 

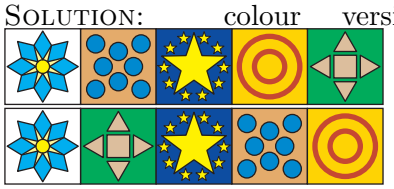
(B) 

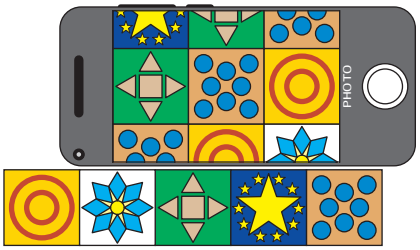
(C) 

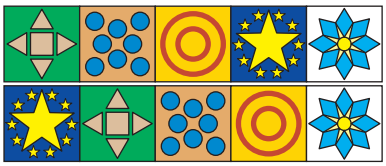
(D) 

(E) 

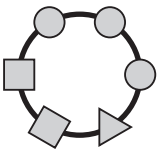
Solution: colour version:

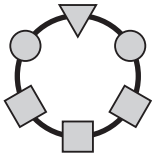


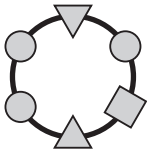


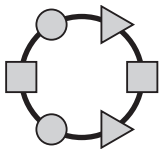


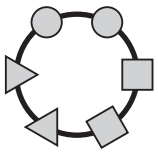
2 (Germany). Luna's bracelet is made of three different types of bead. There are two beads that are spheres next to each other. There are not two beads that are cubes next to each other. Which of the following could be Luna's bracelet?

(A) 

(B) 

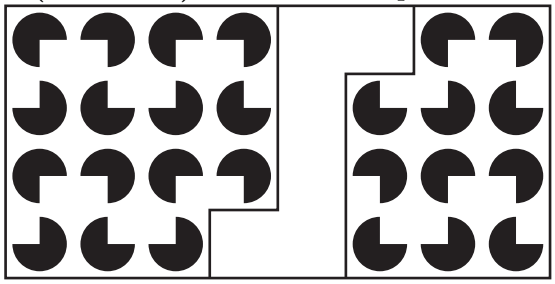
(C) 

(D) 

(E) 

Solution: Only the bracelets (A), (C) and (E) have two sphere beads next to each other. And only the bracelets (C) and (D) have no two cube beads next to each other. Of these bracelets, only bracelet (C) can be Luna's bracelet.

3 (Denmark). Which of the pieces shown below is needed to complete the puzzle?



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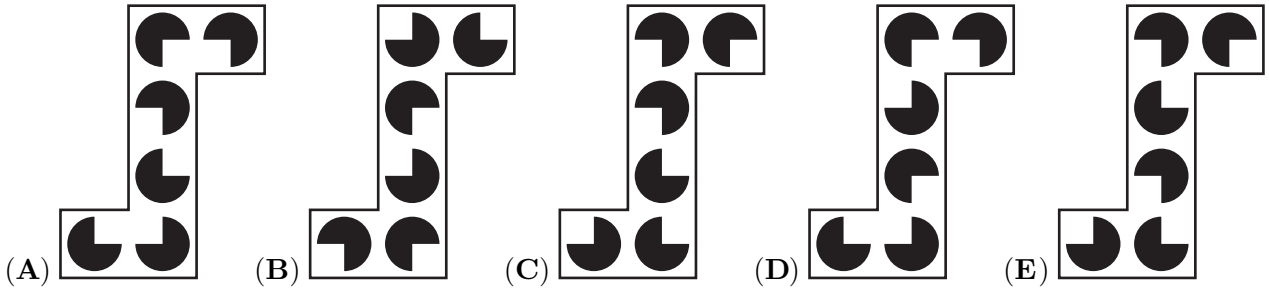
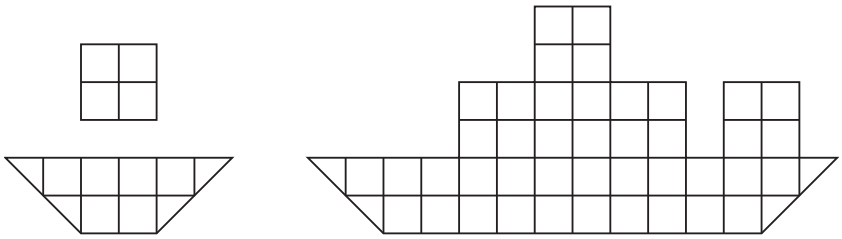


(**A**) (**B**) (**C**) (**D**) (**E**)

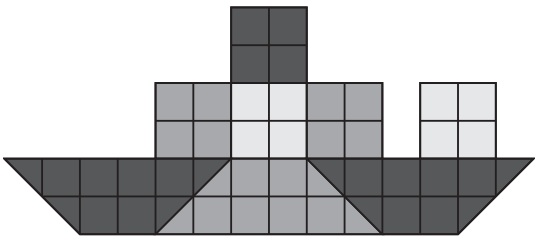
Solution:

4 (Iran). Sepehr has two types of small paper pieces, as shown. How many small pieces of paper in total does he need to create the ship shown?



(**A**) 4 (**B**) 5 (**C**) 6 (**D**) 7 (**E**) 8

Solution:



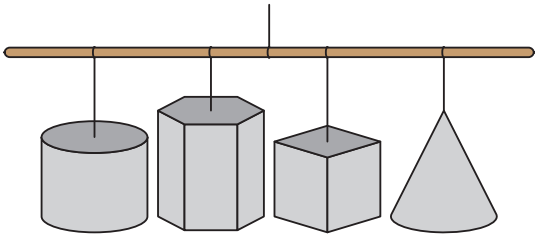
5 (Brazil). A standard dice has six faces numbered from 1 to 6. The sum of the numbers on opposite faces is always 7. The numbers on three faces that share a common vertex have a sum of 14. What are the numbers on the other three faces?

(**A**) 1, 2 and 4 (**B**) 3, 5 and 6 (**C**) 2, 5 and 6 (**D**) 1, 2 and 6 (**E**) 2, 3 and 4

Solution: The sum of all numbers on the die is $1 + 2 + 3 + 4 + 5 + 6 = 21$, so the sum of the three numbers on the other faces is $21 - 14 = 7$. Therefore, those numbers are 1, 2, and 4.

SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST

6 (Norway). There are four solids hanging in the classroom, as shown in the picture. Betty is looking at them from below. What can she see?



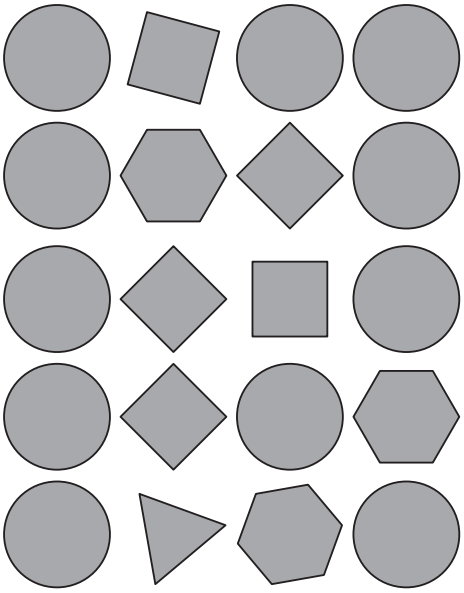
(**A**)

(**B**)

(**C**)

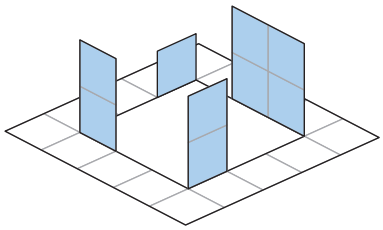
(**D**)

(**E**)

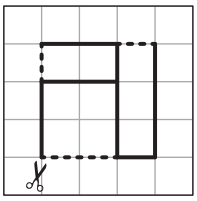


SOLUTION: The picture of answer B after all the changes

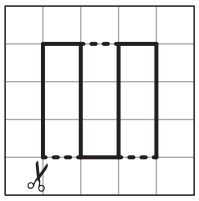
7 (Norway). On a template, dashed lines show where to fold and solid lines show where to cut. Which of the templates below did Edvard use to create the figure on the right?



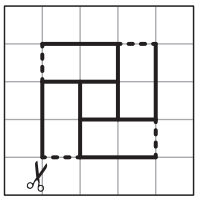
(**A**)



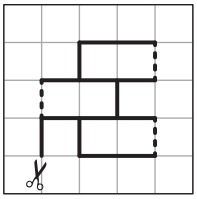
(**B**)



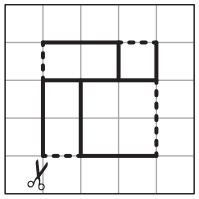
(**C**)



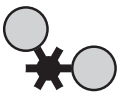
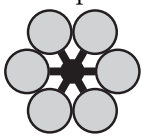
(**D**)



(**E**)



SOLUTION:

8 (Germany). Anthea wants to lay several templates of the following shape  on top of each other to form a flower that looks like this: 
The templates can overlap. What is the smallest number of templates she needs?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

SOLUTION: To fill the single empty spot of the first template, Anthea has to put a template with a circle on this spot. Then it looks like this:

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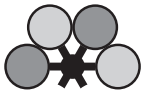


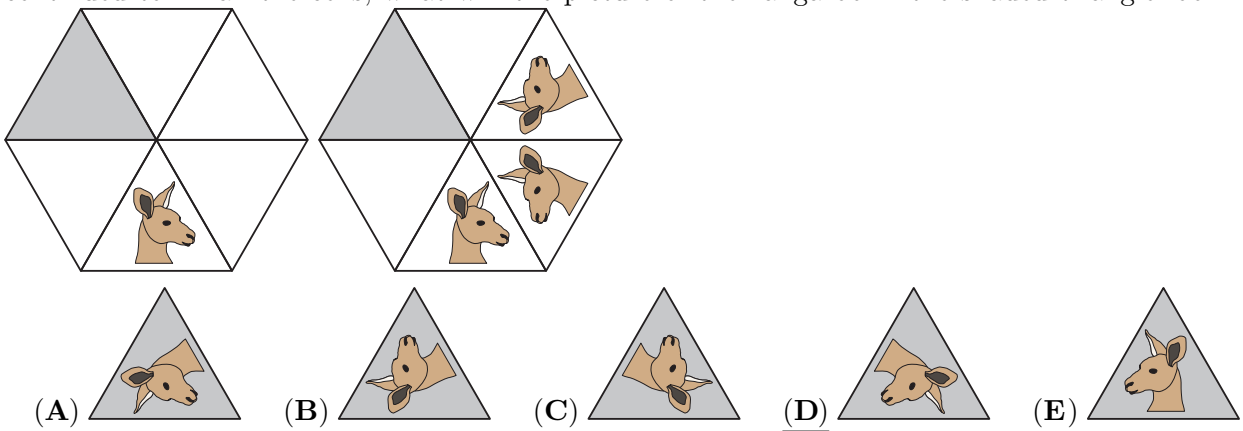
Now there remain two empty spots that are neighboured. Hence, one more template is not enough, she needs at least 4 templates in total. And it is easy to find examples with 4 template.

9 (Ghana). A pizza is cut into 8 equal slices. Max eats $\frac{1}{4}$ of the pizza and Grace eats $\frac{1}{2}$ of what is left. How many slices remain?

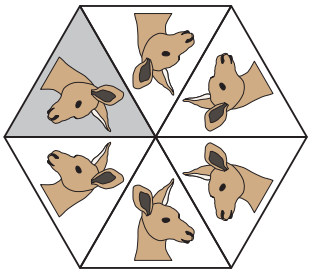
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

SOLUTION: Max eats $\frac{1}{4}$ of 8 slices, that means 2 slices. Then $8 - 2 = 6$ slices are left. Grace eats $\frac{1}{2}$ of these 6 slices, that means 3 slices. Hence, $6 - 3 = 3$ slices remain.

10 (Mexico). The first figure shows the face of a kangaroo. The second figure shows what has happened after the kangaroo has been reflected twice in the lines of the figure. When this process is continued to fill all the cells, what will the picture of the kangaroo in the shaded triangle look like?



SOLUTION: The complete reflection is shown in the figure below.



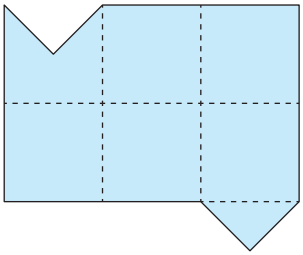
4 points

11 (Slovakia). Cave tours are conducted in three-seater vehicles. The vehicles leave at two-minute intervals, and the ride takes 10 minutes. The first group of three from a large group of 30 people started their tour at 13:00. At what time did the final group of three from the large group finish their tour?

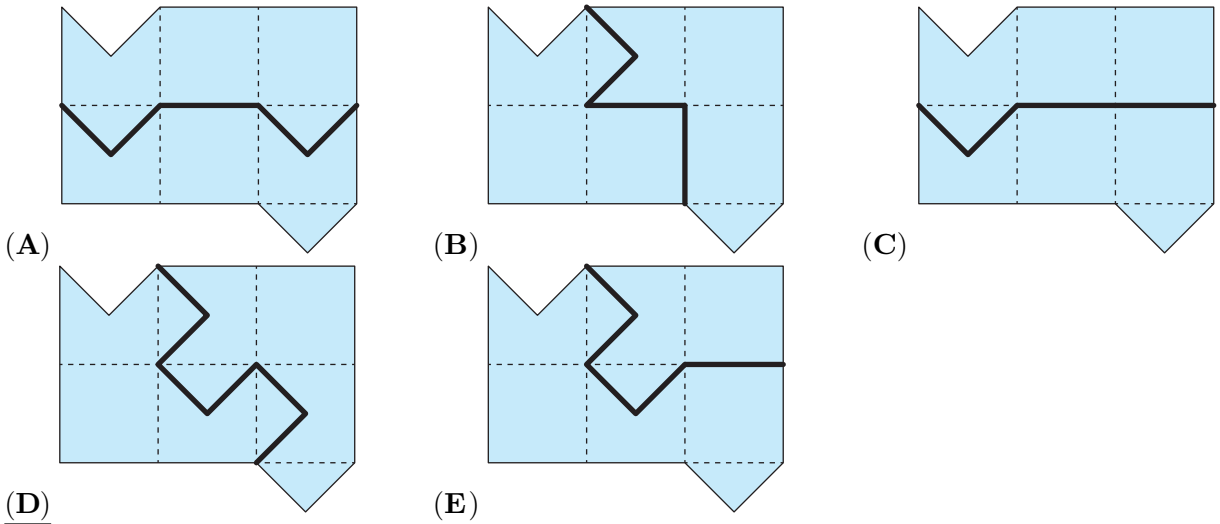
(**A**) 13:18 (**B**) 13:20 (**C**) 13:28 (**D**) 13:30 (**E**) 14:40

SOLUTION: the 10th vehicle leaves at 13.18 and it takes 10 minutes to arrive. So the answer is 13.28

SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST



12 (United States). Which option shows a cut that divides the figure shown into two identical parts? The parts may be flipped.



SOLUTION: Only Option D divides the figure into two identical parts.

In A and E, the two parts are not the same size.

In B and C, the two parts are not the same shape.

Therefore, D is correct.

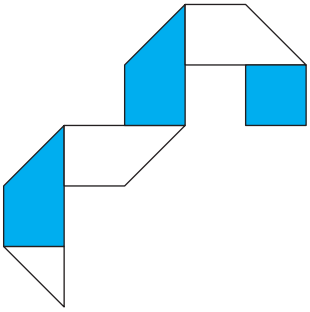
13 (United States). A 24-hour digital clock runs correctly but the positions of two of its digits are swapped. The clock currently shows 15:69. What will the clock show 1 minute later?

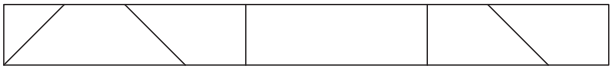
(**A**) 10:70 (**B**) 15:70 (**C**) 16:69 (**D**) 16:70 (**E**) 25:69

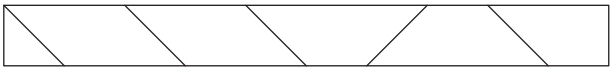
SOLUTION: At the current time, 15:69, the digit 6 must be swapped because a 6 can never appear in the tens place of the minutes. The correct digit in that position must be less than 6, so the real time is either 65:19 or 16:59. Since 65:19 is not valid, our only option is 16:59, which means the digits 5 and 6 are swapped. One minute later, the actual time will be 17:00. Swapping the digits back gives 10:70.

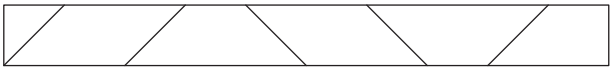
SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST

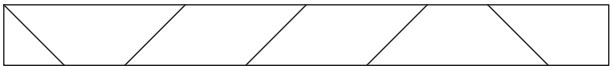
14 (Poland). Lukas has a rectangular strip of paper. One side is white, the other side is dark. On the white side he drew 5 lines and folded the strip along them, as shown in the picture. What did the strip look like before folding?

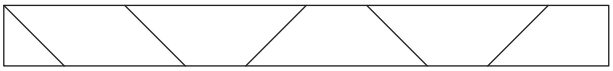


(**A**) 

(**B**) 

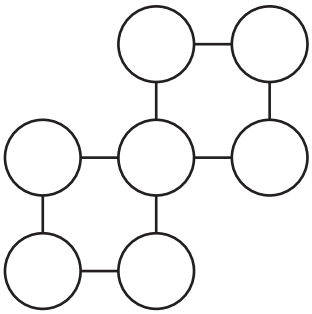
(**C**) 

(**D**) 

(**E**) 

Solution: L.M. There are five fold lines on the strip. Observe that the three middle ones are parallel. Therefore, only answer D can be correct.

15 (Poland). The numbers 0, 1, 2, 3, 4, 5 and 6 are written in the circles shown on the right. Each number is placed in a different circle, so that the sum of the numbers in each row is the same. What is the product of the numbers written in the middle row?



(**A**) 0

(**B**) 15

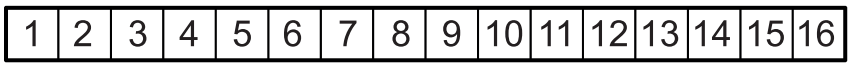
(**C**) 18

(**D**) 24

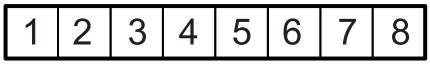
(**E**) 30

Solution: The rows should have the same sum, the number zero must be placed in the middle row. Since the product in the middle row involves the zero, the product always will be zero. One example is 5+2=7, 6+0+1=7, 3+4=7.

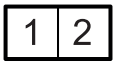
16 (Russia). Ira wrote the numbers from 1 to 16 into the cells of a strip of paper, as shown.



Then she folded the strip in half, as shown:



She continued folding it in half in the same way and ended up with only two cells:



Ira then poked a needle through the whole strip where the number 1 was written, unfolded the strip and added up all numbers in pierced cells. What answer did she get?

(**A**) 64 (**B**) 68 (**C**) 99 (**D**) 128 (**E**) 136

Solution: To solve this there is no need to summarize all the marked numbers. It is enough to notice that the strip is folded each time symmetrically around the middle.
After the first fold opposite each number (denote x) is the number $17 - x$ and the sum of each pair is 17 ($16 + 1$, $15 + 2$ etc.). Mentally glue the strip and 'replace' it with a single-layer strip 8 cells long with the number 17 written in each cell.

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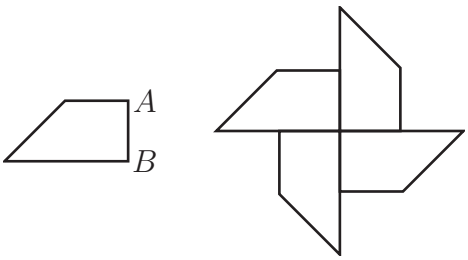
Now it is clear that after the next fold the sum of the 'glued' numbers will double. Again mentally replace it with a 4-cell strip with the number 34 written in each cell. After folding and mentally replacing it again we are left with a strip of two cells each of which has the number 68. This is the sum of the numbers that will be in the cells pierced by the needle.

17 (Philippines). The trapezium on the left has a perimeter of 22 cm. Four of these trapezia are joined together, without overlapping, to form the pinwheel design shown on the right. The perimeter of the pinwheel is 56 cm. What is the length of the side AB in the trapezium?



(**A**) 8 cm (**B**) 6 cm (**C**) 3 cm
(**D**) 4 cm (**E**) 5 cm

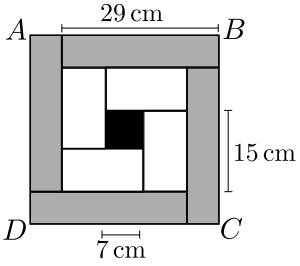
SOLUTION: Perimeter of one trapezoid = 22cm
Four such trapezoids separately would have total perimeter $4 \times 22 = 88$.
When arranged as in Figure 2, some sides become internal (shared) edges.
In this pinwheel, exactly four edges are shared: the vertical and the horizontal central segments, each equal to side AB of the trapezoid. Hence the total shared length is 4AB.
When two polygons share an edge of length L, the combined perimeter decreases by 2L. Reduction $= 4x + 4x = 8x$. Set up: $88\text{cm} - 8x = 56\text{cm}$, so $8x = 32\text{cm}$, and hence $x = 4\text{cm}$. Therefore, the answer is (D), 4cm.

18 (Italy). There are some toffees in a box. Charles, Paul and Simon take turns to take some toffees from the box. Charles takes 1, then Paul takes 2, then Simon takes 3, then Charles takes 4, then Paul takes 5 and so on. When the box does not contain enough toffees to follow this rule, the person whose turn it is takes all the remaining toffees. Paul took 25 toffees in total. How many toffees were there in the box initially?

(**A**) 48 (**B**) 50 (**C**) 55 (**D**) 56 (**E**) 65

SOLUTION: Until there are enough toffees in the box, Charles can take in turn 1, 4, 7, 10, 13,... toffees, Paul 2, 5, 8, 11,..., Simon 3, 6, 9,... . So the possible amounts for Paul are 2, 7, 15, 26,... . Paul getting only $25 = 2 + 5 + 8 + 10$ toffees means that Paul was the last boy to move and took only 10 instead of 11 toffees: hence the initial number of toffees in the box $1 + 2 + 3 + ... + 10 + 10 = 65$.

19 (Mexico). The square $ABCD$ is divided into 4 identical grey rectangles, 4 identical white rectangles and one black square, as shown. The side-length of the black square is 7 cm. The side-length of the longer sides of the white rectangles is 15 cm and the side-length of the longer sides of the grey rectangles is 29 cm. What is the side-length of the square $ABCD$?



(**A**) 33 cm (**B**) 34 cm (**C**) 35 cm
(**D**) 36 cm (**E**) 37 cm

SOLUTION: The shortest side of each white rectangle is $15 - 7 = 8$ cm. So the shortest side of each gray rectangle measures $29 - (8 + 15) = 6$ cm. Thus $|AB| = 29 + 6 = 35$ cm.

20 (Iran). A group of students has a box of apples. They want to divide the apples equally among themselves. They note that:
If there were 80 more apples, every student would get 4 more apples each.
If there were 8 fewer students, every student would get 6 more apples each.
How many apples are in the box?

(**A**) 240 (**B**) 180 (**C**) 160 (**D**) 120

SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST

(**E**) It can not be determined.

SOLUTION: Number of students equals to: $80/4 = 20$ students. Then we have $20-8 = 12$, $12 \times 6 = 72$, $72/8 = 9$. So each student originally received 9 apples. Total number of apples equals to $20 \times 9 = 180$.

5 points

21 (United States). A detective is trying to determine the route the suspect took. The suspect gives three different statements:

"I went from New York via Chicago to Omaha."

"I went from New York via Miami to Kansas City."

"I went from San Francisco via Miami to Omaha."

In each statement, exactly one of the places and its position in that statement is correct. What is the route the suspect took?

(**A**) New York → Chicago → Omaha

(**B**) San Francisco → Chicago → Kansas City

(**C**) New York → Miami → Kansas City

(**D**) San Francisco → Miami → Omaha

(**E**) Chicago → San Francisco → Kansas City

SOLUTION: We know: Each statement has exactly one correct city in the correct position. The other two cities in that statement are incorrect (either the wrong city or the wrong position).

Step 1. First city: San Francisco

In the first two statements, the first city is New York. Since only one item in each statement can be correct, New York cannot be the first. In the third one, he says San Francisco first. This must be the correct first city.

Step 2. Second city: Chicago

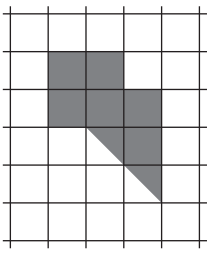
In the first statement, the second city is Chicago. In the second and third statement it is Miami. Since only one item in each statement can be correct, Miami cannot be second. Thus, the second city is Chicago.

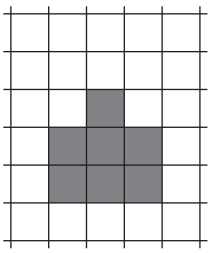
Step 3. Third city: Kansas City

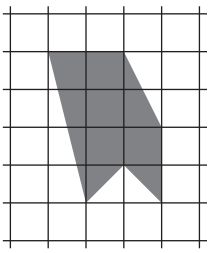
In the first and third statement the third city is Omaha. Since only one item in each statement can be correct, Omaha cannot be third. Third city in the second statement, Kansas City, must be correct.

The suspect's actual route was San Francisco → Chicago → Kansas City.

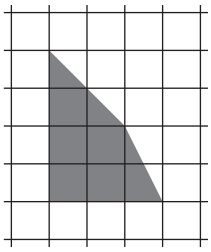
22 (Denmark). The areas of four of the shaded regions shown are the same. Which shaded region has a different area?

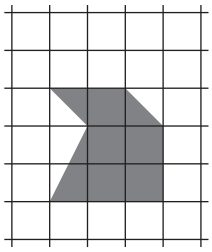
(**A**) 

(**B**) 

(**C**) 

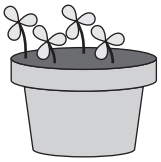
SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST

(**D**) 

(**E**) 

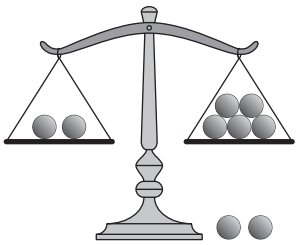
SOLUTION:

23 (Germany). Five siblings have each planted flowers in a pot on their balcony. Now the first flowers are starting to sprout. The 5 pots are shown below. In Jim's and Frederic's pots there are 3 times as many flowers in total as there are in Zoe's pot. In Frederic's and Carl's pots there are twice as many flowers in total as there are in Rene's pot. Which is Frederic's pot?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

SOLUTION: Jim and Frederic together have at most $4+5=9$ sprouts, so Zoe has at most $9:3=3$ sprouts, that means 1, 2 or 3.
If Zoe has 1, then Jim and Frederic will have 1 and 2. This is not possible since there is only one pot with 1 sprout.
If Zoe has 3, then Jim and Frederic will have 4 and 5. Then Rene must have either 1 or 2, and hence Frederic and Carl together have either 2 or 4. This is also not possible since Frederic alone has 4 or 5. Therefore, Zoe has 2 and Jim and Frederic have 1 and 5 in some order. Carl must have an odd number, because Frederic and Carl together have an even number. So Carl has 3 and Rene has 4. And finally, Frederic has $2\cdot 4-3=5$.

24 (Brazil). Julia has 9 balls with masses 1kg, 2kg and so on up to 9kg. She puts seven of the balls on a scale so that the scales balance, as shown. Two of the balls are placed on the left plate and five of the balls are placed on the right plate. What is the smallest possible total of the masses of the two balls that are not used?



(**A**) 5 kg (**B**) 7 kg (**C**) 9 kg (**D**) 11 kg (**E**) 17 kg

SOLUTION: The 2 balls on the left weights the same as the 5 balls on the right. So, the the weight's sum of these 7 balls must be the largest possible, in order to the weight's sum of the 2 balls taken off be the minimum possible. Consequently, the weight's sum of the 2 balls on the left plate must be the largest. We can try keep the balls 9 and 8, with sum 17 in the plate. Then the sum of the five balls on the right plate is 17 , and the sum of the 2 balls taken off is $45-2\times 17=45-34=11$ In fact this is possible, because $1+2+3+4+7=17$ and $5+6=11$

25 (Germany). Flo has a combination lock with 4 digits ranging from 0 to 9. He has forgotten the combination, but he does remember that the digits are all odd and they either increase or decrease from left to right. What is the largest number of combinations he would need to try to be sure he can open his lock?

(**A**) 6 (**B**) 8 (**C**) 10 (**D**) 12 (**E**) 14

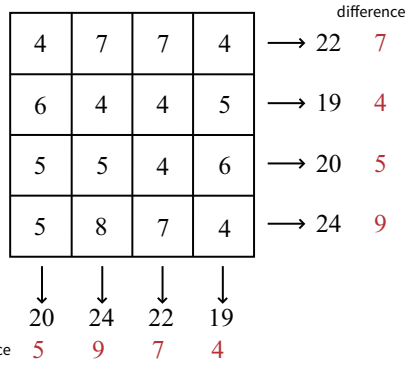
SOLUTION: There are 5 odd digits: 1, 3, 5, 7, 9. Since the combination consists of 4 odd digits, exactly one of the 5 digits 1, 3, 5, 7, 9 is not used. And the digits that are used either increase or decrease from left to right. That makes $5\cdot 2=10$ possible combinations:
3579 or 9753, 1579 or 9751, 1379 or 9731, 1359 or 9531, 1357 or 7531.

SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST

26 (China). Renate removed several numbers from the table below so that the sum of the remaining numbers in each row and each column is 15. What is the sum of the numbers she removed?

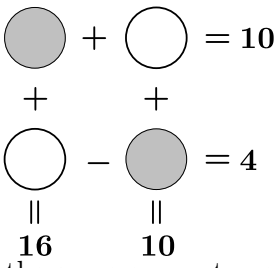
4	7	7	4
6	4	4	5
5	5	4	6
5	8	7	4

(**A**) 31
(**B**) 29
(**C**) 27
(**D**) 25
(**E**) 24



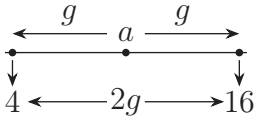
SOLUTION:
Method one: For the difference 9, we know that we need to eliminate 5 and 4, and finally confirm that the third 7 in the first row should also be eliminated. The sum of the eliminated numbers is $5+5+4+4+7=25$.
Method two: Directly add the missing numbers together, which is the sum of the eliminated numbers: $7+4+5+9=25$.
Method three: add up all numbers and substract 4×15.

27 (Catalonia). Each circle contains a number so that the calculations are correct. What is the sum of the numbers in grey circles?



(**A**) 10
(**B**) 12
(**C**) 14
(**D**) 16
(**E**) 23

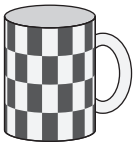
SOLUTION: If we look at the first row and the second column, which add up to the same amount, we will see that the two gray circles must contain the same number. Say it g. Let a be the number in the second row, first column. Let's imagine that we have placed it on a segment. The first column tells us that if we mark a segment of length g to the right of a it ends in 16. If we look at the second row it tells us that if we mark a segment of legth g to the left of a it ends in 4. The distance between the two points that we have marked, that is $16-4=12$ is exactly $2g$, namely the sum of the two gray circles.



28 (Germany). The five cups shown belong to Leonard, Rajesh, Amy, Penny and Sheldon, in some order. All the cups' handles are either black or white. Leonard's cup and Rajesh's cup are the same size but their handles are different colours. Amy's cup and Penny's cup are different sizes but their handles are the same colour.
Which cup belongs to Sheldon?

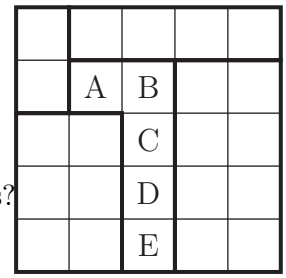
SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST

(**A**)  (**B**)  (**C**) 

(**D**)  (**E**) 

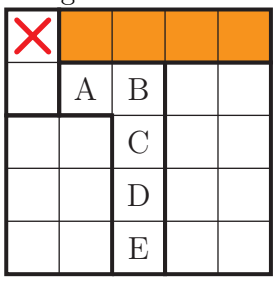
SOLUTION: Since one of Amy's and Penny's cups is small, Leonard's and Rajesh's cups have to be large. Hence, Sheldon's cup is small. Since one of Leonard's and Rajesh's cups has a white handle, Amy's and Penny's cups have black handles. Hence, Sheldon's cup has a white handle. Sheldon's cup is the cup (B).

29 (Iran). A square playground is divided into 25 small squares which form five regions. The regions are marked in the picture with a bold line. Karla the Kangaroo places five swings in the playground. Each row, each column and each region contains exactly one swing. No two swings can occupy adjacent squares. Adjacent means that they share a side or a corner.
In which of the squares marked with a letter can Karla place one of the swings?

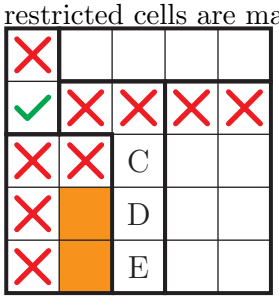


(**A**) A (**B**) B (**C**) C
(**D**) D (**E**) E

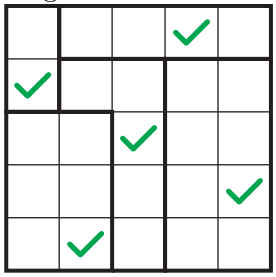
SOLUTION: We must place exactly one swing in the top row of the map. Also, there must be one swing in the colored region. So, the top-left corner cell cannot have a swing.



In the region that has two cells, one of them cannot have a swing. So the swing must be placed in the lower cell, which is marked with a checkmark. As a result, all cells in the same row or column as this cell, as well as all its neighboring cells, cannot have a swing. In the figure below, all of these restricted cells are marked with an X.



In the above diagram, one of the colored cells must contain a swing. But each of these two colored cells is adjacent to both D and E. So if the swing is placed in either of the colored cells, neither D nor E can have a swing. Therefore, the only possible option is to place the swing in cell C. The following diagram shows one valid way to place all the swings:



30 (Russia). Ali wrote out all the numbers from 1 to 7000 in order, without separating them with spaces, commas or any other symbols. How many times does the digit sequence '2026' appear in the resulting list of numbers?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

SOLUTION: The digit sequence '2026' can appear at the boundary of two consecutive numbers:
1. where the first ends in 20 and the next starts with 26 (numbers 2620 and 2621);
2. where the first ends in 202 and the next starts with 6 (numbers 6202 and 6203).
There are no numbers that end in 2 and are followed by a number beginning with 026.
So the sequence '2026' appears at the boundary of: 2620-2621 and 6202-6203, plus the number 2026 itself. That makes a total of 3 times.

SOLUTIONS INCLUDED - DO NOT USE FOR CONTEST